### Problems

1. For each of the following distributions, derive/find all of the following: PMF/PDF, CDF, median,  $\mu$ ,  $\sigma$ ,  $\sigma^2$ . Distributions are: Uniform Distribution, Birthday Distribution, Exponential Distribution, Laplacian Distribution, Pareto Distribution, Normal Distribution, Logistic Distribution, Dart Throw Distribution, Die Throw, Coin Flips, Number of Boys-Girls in a Family, Binomial Distribution.

# Solution: Uniform • PDF: $f(x) = \begin{cases} \frac{1}{b-a} & x \in [a,b] \\ 0 & \text{otherwise.} \end{cases}$ • CDF: $F(x) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & x \in [a,b] \\ 1 & x \ge b \end{cases}$ • Median: $\frac{a+b}{2}$ . • Mean: $\mu = \frac{a+b}{2}$ . • Standard Deviation $\sigma = \frac{b-a}{\sqrt{12}}$ . Birthday • PDF: $f(x) = \begin{cases} \frac{1}{365} & x \in [0, 365] \\ 0 & \text{otherwise.} \end{cases}$ • CDF: $F(x) = \begin{cases} 0 & x \le 0\\ \frac{x}{365} & x \in [0, 365] \\ 1 & x \ge 365 \end{cases}$ • Median: $\frac{365}{2}$ . • Mean: $\mu = \frac{365}{2}$ . • Standard Deviation $\sigma = \frac{365}{\sqrt{12}}$ .

Exponential • PDF:  $f(x) = \begin{cases} \lambda e^{-\lambda x} & x \ge 0\\ 0 & \text{otherwise.} \end{cases}$ • CDF:  $F(x) = \begin{cases} 0 & x \le 0\\ 1 - e^{-\lambda x} & x \ge 0 \end{cases}$ • Median:  $\frac{\ln 2}{\lambda}$ . • Median:  $\mu = \frac{1}{\lambda}$ . • Mean:  $\mu = \frac{1}{\lambda}$ . • Standard Deviation  $\sigma = \frac{1}{\lambda}$ . Laplacian • PDF:  $f(x) = \frac{\theta}{2}e^{-\theta|x|}$ . • CDF:  $F(x) = \begin{cases} \frac{1}{2}e^{\theta x} & x \le 0\\ 1 - \frac{1}{2}e^{-\lambda x} & x \ge 0 \end{cases}$ . • Median: 0. • Mean:  $\mu = 0$ . • Standard Deviation  $\sigma = \frac{\sqrt{2}}{\lambda}$ . Pareto

• PDF: 
$$f(x) = \begin{cases} px^{-p-1} & x \ge 1\\ 0 & \text{otherwise.} \end{cases}$$
  
• CDF:  $F(x) = \begin{cases} 0 & x \le 1\\ 1 - x^{-p} & x \ge 1. \end{cases}$ 

• Median: 
$$\sqrt[p]{2}$$
.

- Mean:  $\mu = \frac{p}{p-1}$  if p > 1, diverges if  $p \le 1$ .
- Standard Deviation  $\sigma = \frac{p}{(p-1)^2(p-2)}$  if p > 2 and diverges if  $p \le 2$ .

Normal

• PDF: 
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/(2\sigma^2)}$$
.

- CDF:  $F(x) = \int_{-\infty}^{x} f(t) dt$ . Use z-score table to look this up.
- Median:  $\mu$
- Mean:  $\mu$ .
- Standard Deviation  $\sigma$ .

#### Logistic

• PDF: 
$$f(x) = \frac{ae^{-ax+b}}{(1+e^{-ax+b})^2}$$

• CDF: 
$$F(x) = \frac{1}{1 + e^{-ax+b}}$$
.

- Median:  $\frac{b}{a}$ .
- Mean:  $\mu = \frac{b}{a}$ .
- Standard Deviation  $\sigma = \frac{\pi^2}{3a^2}$ .

Dart Throw

• PDF: 
$$f(x) = \begin{cases} \frac{2x}{c^2} & 0 \le x \le c\\ 0 & \text{otherwise.} \end{cases}$$
  
• CDF:  $F(x) = \begin{cases} 0 & x \le 0\\ \frac{x^2}{c^2} & 0 \le x \le c\\ 1 & x \ge c \end{cases}$ 

• Median: 
$$\frac{c}{\sqrt{2}}$$
.

• Mean: 
$$\mu = \frac{2a}{3}$$
.

• Standard Deviation  $\sigma = \frac{a}{\sqrt{18}}$ .

#### Die Throw

- PMF:  $f(x) = \frac{1}{6}$  for x = 1, 2, 3, 4, 5, 6.
- CDF: Step function with jumps at 1, 2, 3, 4, 5, 6 of height  $\frac{1}{6}$ .
- Median: 3.5.
- Mean: 3.5.
- Standard Deviation  $\frac{\sqrt{35}}{\sqrt{12}}$ .

Coin Flips=Boys-Girls

- PMF:  $f(k) = \frac{\binom{n}{k}}{2^n}$  for  $0 \le k \le n$ .
- CDF: Step function with jumps at  $0 \le k \le n$  of height  $\frac{\binom{n}{k}}{2^n}$ .
- Median:  $\frac{n}{2}$ .
- Mean:  $\frac{n}{2}$ .
- Standard Deviation  $\frac{\sqrt{n}}{2}$

Binomial

- PMF:  $f(k) = \binom{n}{k} p^k (1-p)^{n-k}$  for  $0 \le k \le n$ .
- CDF: Step function with jumps at  $0 \le k \le n$  of height  $\binom{n}{k} p^k (1-p)^{n-k}$ .
- Median:  $\lfloor np \rfloor$  or  $\lceil np \rceil$ .
- Mean: np.
- Standard Deviation  $\sqrt{np(1-p)}$ .

## True/False

2. True **FALSE** Histograms are always defined over intervals of the form [a, b] with  $a \ge 0$  because probabilities are always non-negative.

**Solution:** The height of the histogram is always  $\geq 0$  but the domain doesn't have to be.

- 3. **TRUE** False The height of a rectangle in a histogram equals  $\frac{\text{the amount of the data corresponding to the subinterval}}{\text{the width of the subinterval}}$ , because all rectangular areas in a histogram must sum up to 1.
- 4. True **FALSE**  $P(x) = e^{-x^2}$  is a PDF on  $\mathbb{R}$ .

**Solution:** The area underneath the curve is  $\sqrt{\pi}$  so it is not a PDF. We need to normalize it by  $\frac{1}{\sqrt{\pi}}$  to make it one.

5. True **FALSE** Uniform PDFs are defined by f(x) = c for all  $x \in \mathbb{R}$ .

**Solution:** All uniform PDFs must be finite on a domain like [a, b].

6. True **FALSE** Shifting the bell-shaped PDF  $f(x) = \frac{1}{\sqrt{\pi}}e^{-x^2}$  to the left by 5 units results in another PDF  $g(x) = \frac{1}{\sqrt{\pi}}e^{-(x+5)^2}$  centered at x = 5.

**Solution:** Shifting to the left results in a PDF centered at x = -5.

- 7. True **FALSE** CDFs are continuous functions on  $\mathbb{R}$  but PDFs could be piecewise continuous on  $\mathbb{R}$ .
- 8. True **FALSE** For a PDF to be centered at x = a, it means that a is the median of the CDF.

Solution: For it to be centered, it needs to be symmetric as well.

9. True **FALSE** We adjusted the solution  $y(t) = y_0 e^{-ct}$  of the exponential decay DE  $y'(t) = -cy(t), y(0) = y_0$  with rate constant c to the PDF  $Y(t) = ce^{-ct}$  for  $x \ge 0$  and y(t) = 0 for x < 0 because we wanted to simplify the initial condition  $y_0$  to match c.

**Solution:** We did this to normalize the PDF to have area 1.

10. **TRUE** False CDFs behave in general like antiderivaties of their PDFs, but there are some situations where the CDF is not continuous and hence there is no way it can be an antiderivative of a PDF.

**Solution:** CDFs could arise from PMFs and hence they are not antiderivatives of a PDF.

11. **TRUE** False The formula  $P(a \le X \le b) = F(b) - F(a)$  for a CDF F(x) works because F(x) can be essentially thoughout of as the "area-so-far" function for a PDF.

12. True **FALSE** To prove that a function F(x) on  $\mathbb{R}$  is a CDF, we need only to confirm that it is non-decreasing on  $\mathbb{R}$ , attains only non-negative values, and that its value tends to 1 and 0 as  $x \to \infty$  and  $x \to -\infty$  correspondingly.

**Solution:** We also need to show that it is right continuous.

13. **TRUE** False The CDF of the bell-shaped PDF  $f(x) = \frac{1}{\sqrt{\pi}}e^{-x^2}$  has a graph that is increasing and looks like a solution to the classic logistic model with carrying capacity K = 1, but it cannot be exactly equal to it because there is no elementary function of an antiderivative of  $e^{-x^2}$  yet we have derived the formula  $g(x) = \frac{1}{1+Ae^{-kt}}$  for these logistic model solutions.

**Solution:** The derivative of the CDF will yield the PDF and so g(x) cannot possibly be an antiderivative of f(x).

- 14. **TRUE** False There are at least three ways to compute the probability of a person to be born in May: using a discrete random variable, and using the PDF or the CDF of a continuous random variable.
- 15. **TRUE** False The formula for the mean of a continuous random variable is a limit version of the mean for a discrete random variable; but while the latter always exists for a finite amount of data, the former may not exist for certain continuous random variables.

Solution: The former may diverge.

- 16. **TRUE** False If the mean is larger than the median, the distribution tends to be more spread away on the right and more clustered together on the left.
- 17. **TRUE** False Both the median and the mean of an exponential distribution directly depend on the initial condition f(0) of the DE f'(x) = Cf(x) and on nothing else.

**Solution:** If the initial value is f(0) = c, then you can show that the mean is 1/c and the median is  $\frac{\ln 2}{c}$ .

18. True **FALSE** During the process of drug decay (or extinction of species), it takes longer for half of the drug (or species) to be gone than for a randomly chosen average molecule (species) to be gone.

**Solution:** Half the drug is gone in  $\frac{\ln 2}{c}$  whereas a random chosen molecule will be gone in  $\frac{1}{c}$ .

19. True **FALSE** If we make the area between a PDF and the x-axis out of uniform cardboard material and make an infinite seesaw out of the x-axis, the point on the x-axis where the seesaw will balance is the median of the distribution because there is an equal material to the left and to the right of the median.

**Solution:** The balancing point is the mean.

- 20. **TRUE** False Exam distributions of large classes tend to have smaller means than medians when the medians are higher than 50% of the maximum possible score.
- 21. True **FALSE** The Pareto distribution fails to have a well-defined mean when the constant  $a \ge 2$ .

**Solution:** It fails to have a well defined mean when  $a \leq 2$ .

22. True **FALSE** Improper integrals resurface when we want to compute probabilities of discrete random variables with finitely many values.

**Solution:** The resurface when we want to calculate the mean and standard deviation of continuous random variables.

23. True **FALSE** For a symmetric distribution, we do not have to calculate the mean because it will always equal the median.

Solution: The mean may fail to exist, but if it does, it will equal the median.

- 24. True **FALSE** Integration by Parts and Substitution Rule can be safely forgotten as Statistics uses ready formulas from Mathematics in all basic examples.
- 25. **TRUE** False The formula for the standard deviation of a continuous random variable is a limit version of the standard deviation for a discrete random variable; but while the latter always exists for a finite amount of data, the former may not exist for certain continuous random variables.

**Solution:** The former doesn't exist in something like  $\frac{1}{x^2}$  for  $x \ge 1$ .

26. True **FALSE** If the standard deviation is larger than the mean for an exam distribution with only non-negative scores, then Chebychev's inequality will for sure predict negative scores on this exam.

**Solution:** Chebychev's inequality will say that the area in a range must be greater than or equal to  $1 - 1/k^2$ , but it doesn't say how it is distributed, so it could be distributed all on the positive side.

27. **TRUE** False All of the median, mean, and standard deviation for an exponential distribution and for the Laplacian distribution directly depend on the initial condition f(0) of the PDF and on nothing else.

**Solution:** The median, mean, standard deviation all depend on  $f(0) = \lambda$ .

28. **TRUE** False The  $\sqrt{12}$  that appears in the denominator of the standard deviation for the uniform birthday distribution will also appear in the denominator of  $\sigma$  for any uniform distribution.

**Solution:** The standard deviation of a uniform distribution on [a, b] is  $\frac{b-a}{\sqrt{12}}$ .

29. True **FALSE** If we make the area between a PDF and the x-axis out of uniform cardboard material, more than 55% of the material will be dedicated to the interval within  $1.5\mu$  of  $\sigma$ 

**Solution:** To use Chebyshev's inequality, we are talking about being within  $1.5\sigma$  of  $\mu$ , not the other way around.

30. True **FALSE** For exam distributions of large classes statisticians use a modified formula for  $\sigma$ , essentially "pretending" that there is one more sample item, in order to get a more accurate "unbiased" statistics. **Solution:** Statisticians use a formula pretending that there is one less sample item, not more.

31. **TRUE** False Improper integrals resurface when we want to justify the shortcut formula for  $\sigma^2$ .

**Solution:** We need to deal with the actual formula for  $\sigma^2$  in terms of integrals to show this.

32. True **FALSE** For a symmetric distribution centered at 0, we do not have to calculate  $\sigma$  because it will always be 0 or not well-defined.

**Solution:** The standard deviation will very much not be 0! And also, it may not be defined.

33. **TRUE** False For any continuous or discrete random variable X with a well-defined mean and variance, it is true that  $\sigma^2(X) + \mu^2(X) = \mu(X^2)$ , where  $X^2$  is a random variable whose values are the squares of the corresponding values of X.

**Solution:** This is because  $Var(X) = E[X^2] - E[X]^2$ .

34. True **FALSE** The PDF of the logistic distribution is given by the solution to the logistic DE y'(t) = ry(t)(1 - y(t)).

Solution: The CDF is given by a solution to that logistic DE.

- 35. **TRUE** False When we increase the initial condition y(0) in the logistic model but keep everything else the same, the logistic distribution tends to move to the left; and similarly, when we increase r, the center of the logistic distribution tends to move to the left.
- 36. True **FALSE** After we establish that the logistic distribution is symmetric about x = a/r and that the normal distribution is symmetric about  $x = \mu$ , we can automatically conclude that their means are correspondingly x = a/r and  $\mu$  without any further integration considerations.

Solution: We need to make sure that the integral converges.

37. True **FALSE** The log-plot transforms the points on a logistic distribution to lie on a line.

**Solution:** We need to change the solutions first by applying F/(1-F).

38. **TRUE** False  $\sigma$  appears twice in denominators in the formula for normal distribution.

**Solution:** It appears once in the denominator of the constant in front, and once in the exponent. The formula is  $\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}$ 

39. **TRUE** False  $\frac{1}{\sqrt{2\pi}}$  ensures that the formula for the normal distribution indeed represents a valid PDF.

Solution: It normalizes the area to 1.

- 40. **TRUE** False The z-scores are made possible by a linear change of variables that converts any normal distribution into the standard normal distribution; and hence, to know everything about normal distributions it suffices to study only the standard normal distribution.
- 41. True **FALSE** z scores are not suitable for computing probabilities of the type  $P(-\infty \le X \le a)$  or  $P(b \le X \le \mu)$  for arbitrary normal distributions.

**Solution:** Because the normal distribution is symmetric around  $\mu$ , we can flip these to the positive side to and do the calculation as before.

42. True **FALSE** Normal distributions are defined only for positive X; yet, when converted to the standard normal distribution, they may be defined for negative X too.

**Solution:** Normal distributions are defined for all X.

43. **TRUE** False  $\pi^2 = \int_{-\infty}^{\infty} \frac{3t^2 e^{-t}}{(1+e^{-t})^2} dt$ .

Solution: This is from the variance of a logistic distribution.

44. **TRUE** False PMFs replace PDFs when moving from continuous to discrete random variables.

Solution: PMFs are the discrete version of a PDF.

45. True **FALSE** CDFs can be defined by the same probability formula for both discrete and continuous variables; however, at the next step when actually computing the CDFs, one must be careful to use correspondingly PMFs with integrals and PDFs with appropriate summations.

Solution: PMFs are with a summation and PDFs are with integrals.

46. True **FALSE** The target spaces for PDFs, PMFs, and probability functions are all the same.

**Solution:** The target space for PDFs and PMFs are  $[0, \infty)$  and for probability functions [0, 1].

47. True **FALSE** The domains of PDFs and PMFs are the corresponding outcome spaces  $\Omega$ .

**Solution:** The domains are  $\mathbb{R}$ .

48. **TRUE** False A set with 10 elements has  $2^{10}$  number of subsets, and hence there will be  $2^{10}$  inputs for any probability function P on any outcome space  $\Omega$  with 10 points.

**Solution:** Probability functions take in any subset of  $\Omega$ , so there are  $2^{10}$  inputs.

49. True **FALSE** While we can define a discrete random variable without using a PMF, a continuous random variable has a PDF in its definition.

Solution: A discrete random variable has to have a PMF.

50. True **FALSE**  $P(A \cup B) = P(A) + P(B)$  as long as A and B are independent events in different outcome spaces.

Solution: Addition holds if A and B are non-overlapping.

51. True **FALSE** Every row in Pascal's triangle represents the PMF of some binomial distribution.

**Solution:** Each row represents the binomial coefficients. But, you need to multiply by  $p^k(1-p)^{n-k}$  to get the distribution.

52. **TRUE** False The probability of having 3 boys in a family of 7 children (assuming equal probability of having boys and girls) is equal to the probability of having 4 boys in a family of 7 children.

**Solution:** This is because  $\binom{7}{3}\frac{1}{2^7} = \binom{7}{4}\binom{1}{2^7}$ .

53. True **FALSE** To each probability space we can associate different random variables, but each probability space has a unique probability function.

Solution: A probability space can have different probability functions.

54. True **FALSE** The formula for the mean of a uniform distribution on [a, b] has  $\sqrt{12}$  in the denominator, while the formula for its standard deviation has 2 in the denominator.

**Solution:** It is the opposite. The formula for the mean has a 2 in the denominator and the formula for the standard error has  $\sqrt{12}$  in the denominator.

- 55. **TRUE** False For any R.V. X it is true that E(5X) = 5E(X), E(5 + X) = E(5) + E(X), and E(5X) = E(5)E(X).
- 56. **TRUE** False The shortcut formula for variance  $Var(X) = E[X^2] E^2[X]$  works for both continuous and discrete R.V.s X.

- 57. **TRUE** False The standard deviation tells us how risky (or not risky) it is to play certain games.
- 58. True **FALSE** The size of the outcome space  $\Omega$  for rolling 5 die is 5<sup>6</sup>.

**Solution:** It is  $6^5$  not  $5^6$ .

59. **TRUE** False The PDFs and PMFs play analogous roles in the formulas for mean (expected value) and standard deviation (standard error).

60. True **FALSE** 
$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t e^{-\frac{t^2}{2}} dt = 1$$
 and  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{t^2}{2}} dt = 0.$ 

Solution: The first integral is 0 (the mean) and the second is 1 (total probability).

- 61. **TRUE** False Var(X) can be defined as the expected value of the square of how far X is from its own mean  $E(X) = \mu$ ; i.e.,  $Var(X) = E[(X - E(X))^2]$ , or using an integral formula for continuous X, and as a consequence  $Var(cX) = c^2 Var(X)$  and SE(cX) = |c|SE(X) for any  $c \in \mathbb{R}$ .
- 62. **TRUE** False The binomial coefficient  $\binom{n}{k}$  will not change if we replace k by n k.
- 63. **TRUE** False The binomial coefficients  $\binom{n}{k}$  increase for the first half of the values of k and then they decrease.
- 64. True **FALSE** The formula for the mean of the average  $\overline{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$  of independent identically distributed RV's has  $\sqrt{n}$  in the denominator.

**Solution:** The formula for the standard deviation has a  $\sqrt{n}$  in the denominator.

65. True **FALSE** For any RV's X and Y, it is true that E(5X - 7Y) = 5E(X) - 7E(Y)and E(XY) = E(X)E(Y).

**Solution:** The first statement is true but the latter only holds for independent X, Y.

66. True **FALSE** The formula for the variance Var(X + Y) = Var(X) + Var(Y) works regardless of whether the RV's X and Y are independent or not.

**Solution:** It is only true if X, Y are independent.

67. **TRUE** False To approximate the height of a tall tree (using similar triangles and measurements along the ground – without climbing the tree!) it is better to ask several people to do it independently of each other and then to average their results, than to do it once just by yourself.

**Solution:** By averaging their heights, you will get a smaller standard deviation and smaller variance and more likely to be closer to the actual value.

- 68. **TRUE** False The z-scores can be used to reduce probability calculations on any normal distribution to using a table in the textbook, as long as we are not too many standard deviations  $\sigma$  from the mean  $\mu$  in our original normal distribution.
- 69. True **FALSE** According to the Central Limit Theorem, the normalized distribution  $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$  is the standard normal distribution for n large, where  $\overline{X}$  is the average of n independent, identically distributed variables, each with mean  $\mu$  and standard error  $\sigma/\sqrt{n}$ .

**Solution:** Each variable has standard error  $\sigma$  and the normalized distribution approaches the standard normal distribution, its not equal to it.

- 70. **TRUE** False The Law of Large Numbers can be viewed as part of the Central Limit Theorem.
- 71. **TRUE** False To prove that  $Var(X) = E(X^2) E^2(X)$  for any RV X, one needs to use that  $E(X_1 + X_2) = E(X_1) + E(X_2)$  for some particular dependent RV's  $X_1$  and  $X_2$ .
- 72. True **FALSE** The Maximum Likelihood (M-L) method uses a probabilistic experiment to estimate a real world parameter  $\theta$ , by considering all possible outcomes of the experiment.

Solution: It only considers the outcomes that you have observed.

73. **TRUE** False The formula for the PMF f(k) of the binomial distribution for k successes out of n independent trials is a polynomial of degree n in the probability p of a success for each trial.

**Solution:** The formula is  $\binom{n}{k}p^k(1-p)^{n-k}$ , which is a polynomial in terms of p.

74. True **FALSE** To determine the Maximum Likelihood of a continuous variable, we can write a table for all values of the individual likelihoods  $L(x|\theta)$  and choose the  $\theta$  that yields the largest value of  $L(x|\theta)$ .

**Solution:** We can do this for the discrete version but for the continuous version, we cannot do this.

- 75. **TRUE** False When using a random variable X and one experiment with it to estimate a parameter  $\theta$ , we compare all values of  $L(x|\theta)$  for X = x fixed and  $\theta$  varying.
- 76. **TRUE** False The log-derivative of  $f(\theta) = L(x|\theta)$  can used to shortcut calculations with the M-L method.

**Solution:** The log derivative gives  $\frac{f'(\theta)}{f(\theta)}$  so whenever  $f'(\theta) = 0$ , the log derivative is also 0.

- 77. **TRUE** False The M-L method applied to estimate the standard error in a normal distribution by taking n independent measurements  $x_1, x_2, \ldots, x_n$  leads to the standard deviation formula for these same n measurements.
- 78. **TRUE** False The mean  $\mu$  and standard error  $\sigma$  of a random variable X can be viewed as parameters and hence the M-L method can be applied to estimate them.

**Solution:** We can use the M-L method and doing so will give us the regular formulas for mean and standard deviation.

79. True **FALSE** If you interview *n* random people in the U.S. about their preference to saying "tom-ei-to" v.s. "tom-a-to" and you get *k* of them saying they prefer "tom-ei-to", you can conclude that based on this data alone your best prediction for the fraction of all people in the U.S. who indeed prefer "tom-ei-to" over "tom-a-to" is  $\frac{n}{k}$ .

**Solution:** The proper fraction is  $\frac{k}{n}$ .

80. True **FALSE** In the class examples of binomial distribution for discrete variables and normal distributions for continuous variables, in order to apply the M-L method, it was crucial that we could find the critical points for the corresponding PMF/PDFs.

**Solution:** We only need to find the critical values for PDFs. For discrete R.V.s, we can just write down all the possibilities and take the largest.

81. **TRUE** False The formula  $(\ln(f(\theta)))' = \frac{f'(\theta)}{f(\theta)}$  makes it possible to shift the process from finding critical points for the original  $f(\theta)$  to finding critical points of  $\ln(f(\theta))$ .

Solution: This is why we can use the log derivative for the M-L method.

82. **TRUE** False The "score equation" for a parameter  $\theta$  estimated by the M-L method is essentially setting up the log-derivative of the likelihood function  $f(\theta) = L(x|\theta)$  equal to 0.

**Solution:** The score equation is just the derivative of the log of the likelihood function.

83. **TRUE** False Working with a larger sample of the population will make the "biased" estimate for  $\sigma^2$  less biased.

**Solution:** As n gets larger, dividing by n and n-1 become closer and closer.

84. True **FALSE** The Null hypothesis is a theory that we believe is true.

Solution: We often want to disprove the null hypothesis.

85. **TRUE** False The higher the significance of a test, the higher the probability of rejecting a true Null hypothesis.

Solution: The significance is the probability of rejecting a true null hypothesis.

86. True **FALSE** Adding up the power and the significance of a test yields 1.

**Solution:** The significance is the probability of making a type I error, the power is 1 minus the probability of making a type II error.

87. **TRUE** False A type-2 error made by a road patrol may result in letting drunken drivers continue driving.

**Solution:** A type II error is caused by lack enforcement, or failing to reject a false hypothesis.

- 88. **TRUE** False A criterion is used on data from experiments to accept the Alternative theory or to keep the Null hypothesis.
- 89. **TRUE** False A number of important questions about hypothesis testing can be reformulated eventually about using z-scores.

**Solution:** By CLT, the distribution of any average is approximately normal so we can phrase things in terms of z scores.

- 90. **TRUE** False The significance of a test shows how often, on the average, we can make a Type 1 error.
- 91. **TRUE** False Using two-sided Alternative Hypotheses  $H_1$  may lead to twice as large significance as their one-sided analogs.
- 92. **TRUE** False Deciding on a rejection region lead to making global policies that affect many local decisions.
- 93. True **FALSE** In class we solved at least one problem in finding the power of a test  $1 \beta$ .
- 94. **TRUE** False The p-value of a possible test result r is the probability that the experiment produces a result that is equally or more extreme (towards  $H_1$ ) than r, assuming  $H_0$  is true.